Some of the review problems – find the entire set on my website

ACP Analysis
Final Exam Review

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NO CALCULATORS ALLOWED SECTION
To get full or partial credit on a problem you must show your work.

1. Find the equation for the inverse of $\frac{6}{3x-2}$. $g^{-1}(x) =$ $\frac{6}{3x} + \frac{2}{3}$

$\frac{6}{3y-2} = x$

$6 = x(3y-2)$

$\frac{6}{x} = 3y-2$

$\frac{6}{x} + 2 = 3y$

$\frac{\frac{6}{x}+2}{3} = y$

$y = \frac{6}{3x} + \frac{2}{3}$

OR

$y = \frac{6+2x}{3x}$

2. Evaluate $g\left(g^{-1}(x)\right)$ and $g^{-1}\left(g\ (x)\right)$. Do the results confirm the two functions are inverses? Explain why or why not. Yes! They "undo" eachother.

$g(g^{-1}(x)) = g\left(\frac{6}{3x}+\frac{2}{3}\right) = \frac{6}{3\left(\frac{6}{3x}+\frac{2}{3}\right)-2} = \frac{6}{\left(\frac{6}{x}+2\right)-2} = \frac{6}{\frac{6}{x}} = x$

$g\left(g^{-1}(x)\right) =$ X

$g^{-1}\left(g(x)\right) =$ X

$g^{-1}(g(x)) = g^{-1}\left(\frac{6}{3x-2}\right) = \frac{6}{3\left(\frac{6}{3x-2}\right)} + \frac{2}{3} = \frac{6}{\frac{18}{3x-2}} + \frac{2}{3} = \frac{6(3x-2)}{18} + \frac{2}{3} = \frac{18x-12}{18} + \frac{2}{3} = x - \frac{2}{3} + \frac{2}{3} = x$

3. Given the functions, $c(x) = \sqrt{3x+5}$ and $d(x) = \left|x^2 - 7\right|$ evaluate,

a. $c\left(\frac{4}{3}\right) = \sqrt{3(4/3)+5} = \sqrt{9} = 3$ a. 3

b. $d(-2) = \left|(-2)^2 - 7\right| = |4-7| = |-3| = 3$ b. 3

c. $d\left(c\left(-\frac{4}{3}\right)\right) = d\left(\sqrt{-4+5}\right) = d(1) = |1^2 - 7| = |1-7| = 6$ c. 6

d. $c(d(3))$ d. $\sqrt{11}$

$c(|9-7|) = c(2) = \sqrt{6+5} = \sqrt{11}$

NO CALC

4. Given the function $f(x) = 8 - \frac{1}{2}\sqrt{x+9}$,

a. state the domain. $x+9 \geq 0$ $\quad x \geq -9$ a. $x \geq -9$

b. state the range. b. $y \leq 8$

c. identify the parent. c. $y = \sqrt{x}$

d. identify the numerical values of A, h and k then describe the transformations from the parent to the graph of this related function.

Numerical Value	Description of the transformation(s) from the parent that results in the graph of $r(x)$.
A: $-\frac{1}{2}$	reflects parent over x-axis and vertically compresses parent
h: -9	shifts parent left 9 units
k: 8	shifts parent up 8 units

e. state the coordinates of the x-intercept. e. $(247, 0)$

$0 = 8 - \frac{1}{2}\sqrt{x+9}$ $\quad -8 = -\frac{1}{2}\sqrt{x+9}$ $\quad +16 = \sqrt{x+9}$ $\quad 256 = x+9$ $\quad 247 = x$

f. state the coordinates of the y-intercept. f. $(0, 6.5)$

$f(0) = 8 - \frac{1}{2}\sqrt{0+9} = 8 - \frac{1}{2}(3) = 8 - \frac{3}{2} = 6.5$

$\begin{array}{r} 16 \\ \times 16 \\ \hline 96 \\ 16 \\ \hline 256 \\ -9 \\ \hline 247 \end{array}$

note change

5. Write the equation of a function with the parent $y = x^3$ that has been ~~reflected over the y-axis,~~ shifted left 10 units and shifted down 1/2 of a unit from the parent.

$B = -1 \quad h = -10 \quad k = -\frac{1}{2}$

$y = (x+10)^3 - \frac{1}{2}$

6. Given the function $m(x) = -2(x+8)^2 - 9$, vertex @ $(-8,-9)$. opens ↓

a. state the domain. a. All real #s

b. state the range. b. $y \le -9$

c. identify the parent. c. $y = x^2$

d. identify the numerical values of A, h and k then describe the transformations from the parent to the graph of this related function.

Numerical Value	Description of the transformation(s) from the parent that results in the graph of $r(x)$.
A: -2	Vertically stretches parent by a factor of 2 and reflects over the x-axis
h: -8	Shifts the parent graph left 8 units
k: -9	Shifts the parent graph down 9 units

e. state the coordinates of the x-intercept(s). e. None

f. state the coordinates of the y-intercept. f. $(0,-137)$

$0 = -2(x+8)^2 - 9$

$9 = -2(x+8)^2$

$\frac{-9}{2} = (x+8)^2$

$\pm\sqrt{\frac{-9}{2}} = x+8$

Imaginary, thus no x-intercepts! Which I knew from the vertex @ $(-8,-9)$ and the fact that the parabola will open down, i.e. have a maximum.

$m(0) = -2(0+8)^2 - 9$

$m(0) = -2(64) - 9$

$m(0) = -128 - 9$

$m(0) = -137$

$\therefore (0,-137)$

7. Given the function $f(x) = 2^{x-1} + 3$,

a. state the domain. a. All real #s

b. state the range. b. $y > 3$

c. identify the parent. c. $y = 2^x$

d. identify the numerical values of A, h and k then describe the transformations from the parent to the graph of this related function.

Numerical Value	Description of the transformation(s) from the parent that results in the graph of $r(x)$.
A: 1	None
h: 1	Shifts parent graph right 1 unit
k: 3	Shifts parent graph up 3 units

e. state the coordinates of the x-intercept. e. NONE

$0 = 2^{x-1} + 3 \Rightarrow -3 = 2^{x-1} \Rightarrow$ No solutions

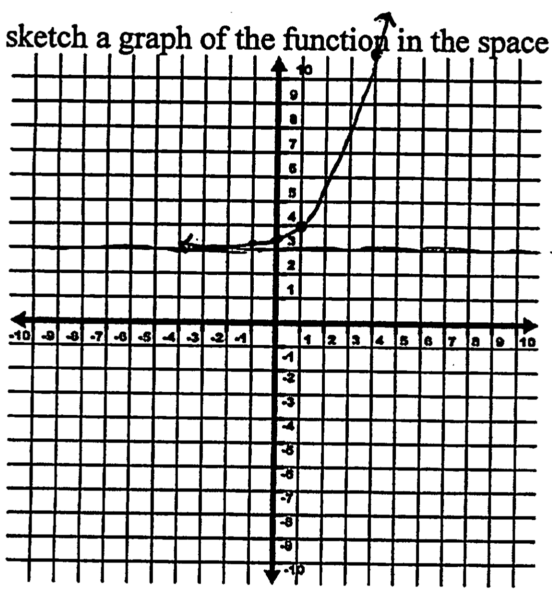
f. state the coordinates of the y-intercept. f. $(0, 3.5)$

$f(0) = 2^{0-1} + 3 = 2^{-1} + 3 = \frac{1}{2} + 3 = 3.5$

g. give the equations of any asymptotes. g. $y = 3$

h. sketch a graph of the function in the space provided:

$Y=3$



x	$f(x)$
0	3.5
1	4
-1	3.25
4	11

NO CALC

8. Given the function $f(x) = \log_2(x-1) - 4$,

   a. state the domain. a. $x > 1$

   $(x-1) > 0$

   $x > 1$

   b. state the range. b. all real #s

   c. identify the parent. c. $y = \log_2 x$

   d. identify the numerical values of A, h and k then describe the transformations from the parent to the graph of this related function.

Numerical Value	Description of the transformation(s) from the parent that results in the graph of $r(x)$.
A: 1	None
h: 1	shifts parent graph right one unit
k: −4	shifts graph down 4 units

   e. state the coordinates of the x-intercept. e. (17, 0)

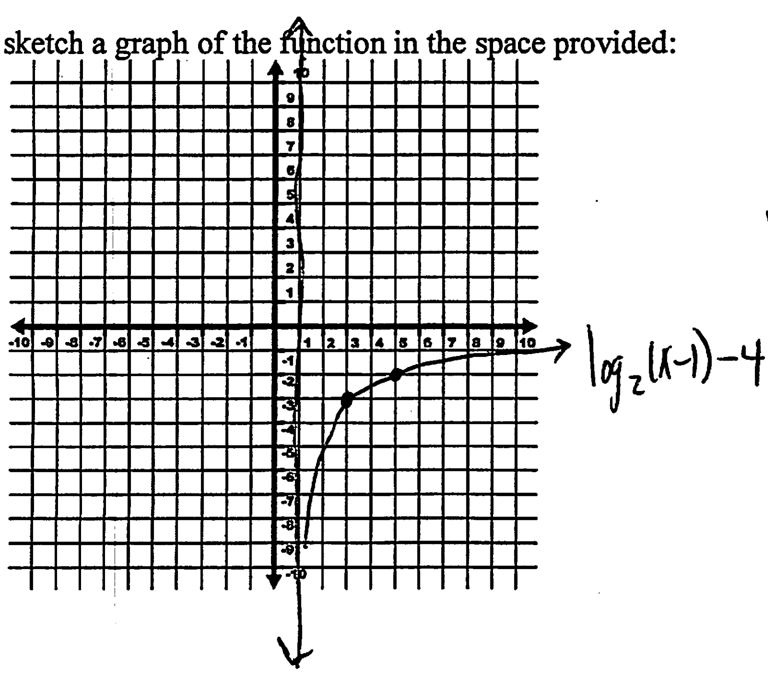
   $0 = \log_2(x-1) - 4$ $4 = \log_2(x-1)$ $2^4 = x-1$ $16 = x-1$ $17 = x$

   f. state the coordinates of the y-intercept. f. No y-intercept

   $f(0) = \log_2(0-1) - 4 \Rightarrow f(0) = \log_2(-1) - 4$ undefined

   g. give the equations of any asymptotes. g. $x = 1$

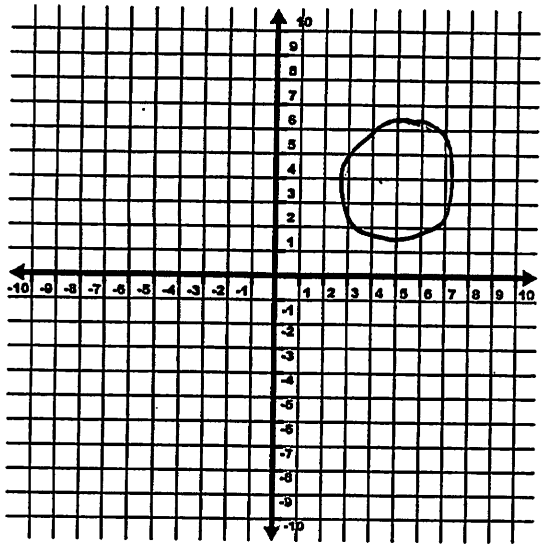
   h. sketch a graph of the function in the space provided:



X	Y
3	$\log_2(3-1) - 4 = 1 - 4 = -3$
5	$\log_2(4) - 4 = -2$

NO CALC

9. On the graph below, draw a relation that is not a function.

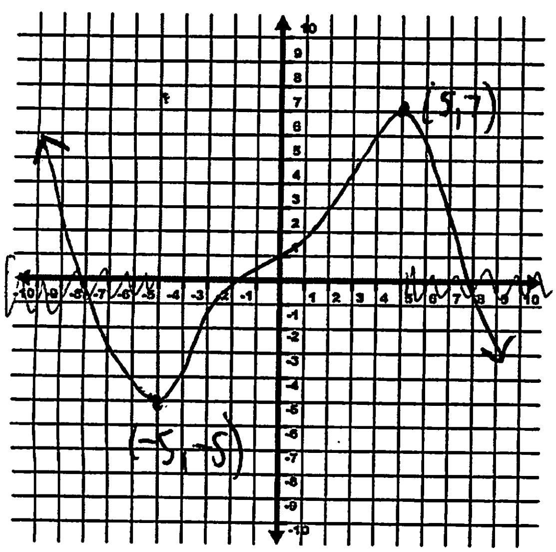


Answers will vary.

Be sure that your graph fails the vertical line test. I.e. there is more than one output for one or more inputs.

10. On the graph below, draw a function that is
- decreasing on the intervals $-\infty < x < -5$ and $5 < x < \infty$
- increasing on the interval $-5 < x < 5$

Label and local minima ox maxima on your sketch



Answers will vary.

Be sure you have a domain of all real #s and a local minimum @ $x=-5$ and a local maximum @ $x=5$.

NO CALC

$$x^2+x=\frac{1}{2} \qquad x=\frac{-1\pm\sqrt{1-4(1)(-\frac{1}{2})}}{2}=\frac{-1\pm\sqrt{3}}{2}$$
$$x^2+x-\frac{1}{2}=0$$

11. Solve the following exponential equations.

a. $4^{1-2x}=2$ $\quad (2^2)^{1-2x}=2^1 \quad 2(1-2x)=1$, $2-4x=1$, $-4x=-1$, $x=1/4$ — a. $x=1/4$

b. $8^{6+3x}=4$ $\quad (2^3)^{6+3x}=2^2 \quad 3(6+3x)=2$, $18+9x=2$, $16=-9x$, $\frac{-16}{9}=x$ — b. $x=-16/9$

c. $3^{x^2+x}=\sqrt{3}$ — c. $x=\frac{-1\pm\sqrt{3}}{2}$

d. $2^{x+1}\bullet 8^{-x}=4$ $\quad 2^{x+1}(2^3)^{-x}=2^2$, $x+1+-3x=2$, $-2x+1=2$, $-2x=1$, $x=\frac{-1}{2}$ — d. $x=-\frac{1}{2}$

e. $9^{2x}=27^{3x-4}$ $\quad 9^{2x}=(3^2)^{2x}=(3^3)^{3x-4} \Rightarrow 4x=9x-12$, $-5x=-12$, $x=12/5$ — e. $x=12/5$

12. Evaluate each of the following.

a. $7^{-2}=\frac{1}{7^2}=\frac{1}{49}$

b. $\log_5 125=x$ $\quad 5^x=125$ $\quad x=3$ $\quad 3$

c. $\log_3\frac{1}{81}$ $\quad \log_3\frac{1}{81}=x$, $3^x=\frac{1}{81}$, $x=-4$ $\quad -4$

d. $\ln e^3$ $\quad 3$

e. $\log 100$ $\quad 2$

13. Given the function $f(x)=\begin{cases} 2x+1, & x\le 0 \\ x^2, & 0<x<3 \\ \log(x-2), & x\ge 3 \end{cases}$, find the value of each.

a. $f(3)$ b. $f(12)$ c. $f\left(-\frac{3}{4}\right)$ d. $f(2)$

$\log(3-2)=\log 1=0$

$\log(12-2)=\log 10=1$

$=2\left(\frac{-3}{4}\right)+1=\frac{-3}{2}+1=-\frac{1}{2}$

$f(2)=2^2=4$

14. Evaluate each expression.

Remember b^x and $\log_b x$ are inverse functions they "undo" each other.

a. $5^{\log_5 12}$ $= 12$

b. $\log_3 3^{2.5}$ $= 2.5$

c. $9^{\log_9 15} - \log_3 3^5$ $= 15 - 5 = 10$

15. Solve for x and check your answers.

a. $2\log_3 x = \log_3 4$

$\log_3 x^2 = \log_3 4 \Rightarrow x^2 = 4$

$x = \pm 2$ But $x > 0$ so $\boxed{x=2}$

b. $\log_7(x^2 - 1) = \log_7 8$

$x^2 - 1 = 8$

$x^2 = 9 \Rightarrow \boxed{x = \pm 3}$

c. $2\log_b(x + 1) = \log_b(-x + 11)$

$\log_b (x+1)^2 = \log_b(-x+11)$

$(x+1)^2 = (-x+11)$

$x^2 + 2x + 1 = -x + 11$

$x^2 + 3x - 10 = 0$

$(x+5)(x-2) = 0$

$x = -5$ or $x = 2$ But $x > -1$ so $\boxed{x=2}$

NO CALC

For each of argument, identify the **main** fallacy, if one exists, from the list below. If there is no fallacy, write "no fallacy." Fallacies may be used more than once or not at all.

List: False cause, slippery slope, appeal to popularity (bandwagon), begging the question (circular reasoning), straw man.

a) The new cell phone policy does not work because as soon as we allow texting in the halls, students will text in class and even during tests.

Slippery Slope

b) We need to have hush-ups on the legs of classroom chairs because the noise is disturbing other classes.

No fallacy - it's a good arguement!

c) The iPhone is the best smartphone. After all, more people have iPhones than any other device.

appeal to popularity

d) People who do not like the new high school are just afraid of change.

Straw man

e) Anyone can stop eating sweets as long as they have the willpower to stop.

Circular Reasoning

f) The traffic in front of the school is worse when it rains.

no fallacy