The Burning Tent Problem

Name(s): ______________________

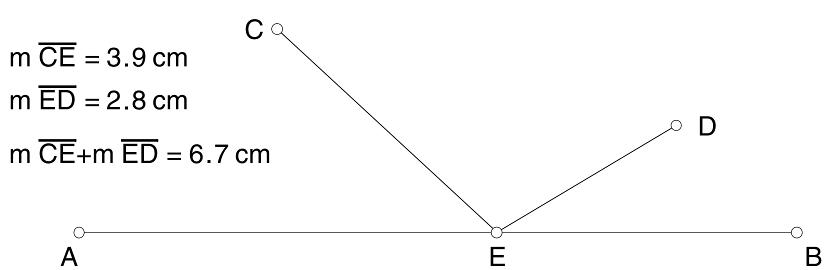
A camper out for a hike is returning to her campsite. The shortest distance between her and her campsite is along a straight line, but as she approaches her campsite, she sees that her tent is on fire! She must run to the river to fill her canteen, then run to her tent to put out the fire. What's the shortest path she can take? The answer to this question is related to the path a pool ball takes when it bounces off a cushion and the path a ray of light takes when it bounces off a mirror. In this activity, you'll investigate the minimal two-part path that goes from a point to a line and then to another point.

Sketch and Investigate

1. Construct a long horizontal segment *AB* to represent the river.
2. Construct points *C* and *D* on the same side of the segment. Point *C* represents the camper and point *D* represents the tent.
3. Construct $\overline{CE}$ and $\overline{ED}$, where point *E* is any point on $\overline{AB}$. These segments together show a path the camper might take running to the river and then to her tent.
4. Measure the lengths of $\overline{CE}$ and $\overline{ED}$.

Choose **Calculate** from the Measure menu to open the Calculator. Click once on a measurement in your sketch to enter it into a calculation.

5. Use the Calculator to find the sum of these two lengths.



To precisely locate point *E*, you might need to change your precision for calculations to thousandths. To do this, in the Edit menu, choose **Preferences**.

6. Drag point *E* and watch the calculated sum change. Move point *E* to the location on the river to which the camper should run.

Select three points that name the angle, with the vertex your middle selection. Then, in the Measure menu, choose **Angle**.

7. Measure the incoming and outgoing angles the camper's path makes with the river (angles *CEA* and *DEB*).

Q1 Once you've found the minimal path, what appears to be true about the incoming angle and the outgoing angle? (See if you can use the angle measures to make the distance sum even shorter.)

So far, you've dragged point E to find an approximate minimal path. Next, you'll discover how to construct such a path.

Double-click the segment.

8. Mark segment AB as a mirror. (Now that it's marked as a mirror, you can stop thinking of it as a river.)

Select the point; then, in the Transform menu, choose **Reflect**.

9. Reflect point D across this segment to create point D'.

10. Construct $\overline{CD'}$ and change its line width to dashed.

Q2 Why is $\overline{CD'}$ the shortest path from point C to point D'?

Q3 Where should point E be located in relation to $\overline{CD'}$ and $\overline{AB}$ so that the sum $CE + ED$ is minimized? Drag point E to test your conjecture.

Explore More

1. In a new sketch, use a reflection to construct a model of the burning tent problem so that the path from the hiker to the river to the tent is always minimal, no matter where you locate the hiker and the tent.

2. What's the shortest mirror you'd need on a wall in order to see your full reflection from your toes to the top of your head? To explore this question, construct a vertical segment representing you and another vertical segment representing a mirror. Construct a point to represent your eye level just below the top endpoint of the segment representing you. Use a reflection to construct the path a ray of light would take from the top of your head to the mirror and to your eye. Construct another path that a ray of light would take from your toes to the mirror to your eye. Adjust the mirror length so that it's just long enough for both light rays to reflect off it. How long is the mirror compared to your height?