

Trigonometric Ratios

Name(s): _____

Right-triangle trigonometry builds on similar-triangle concepts to give you more ways to find unknown measures in triangles. In this activity, you'll learn about trigonometric ratios and how you can use them.

Sketch and Investigate

In steps 1–5, you'll construct a right triangle.

Select point B and \overline{AB} ; then, in the Construct menu, choose **Perpendicular Line**.

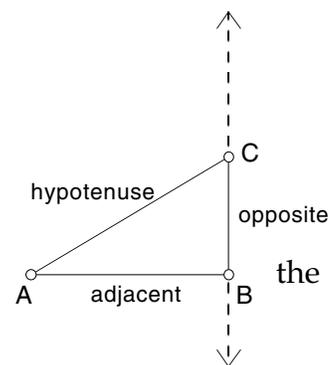
- Construct \overline{AB} .
- Construct a line through point B perpendicular to \overline{AB} .
- Construct \overline{AC} , where point C is a point on perpendicular line.

$$m\angle CAB = 31^\circ$$

$$\frac{m \text{ opposite}}{m \text{ hypotenuse}} = 0.51$$

$$\frac{m \text{ adjacent}}{m \text{ hypotenuse}} = 0.86$$

$$\frac{m \text{ opposite}}{m \text{ adjacent}} = 0.60$$



Using the **Text** tool, click once on a segment to show its label. Double-click the label to change it.

Select, in order, points C , A , and B . Then, in the Measure menu, choose **Angle**.

For each ratio, select the two segments in order. Then, in the Measure menu, choose **Ratio**.

- Hide the line.
- Construct \overline{BC} to finish the right triangle.
- Show the three segments' labels and change the labels to match the figure above right.
- Measure angle CAB .
- Measure the ratios *opposite/hypotenuse*, *adjacent/hypotenuse*, and *opposite/adjacent*.

Q1 Drag point C to change the angles. When the angles change, do the ratios also change?

Q2 Drag point A or point B to scale the triangle. What do you notice about the ratios when the angles don't change? Explain why you think this happens.

Choose **Calculate** from the Measure menu to open the Calculator. In the Functions pop-up menu, choose **sin**.

Click in the sketch on the measure of $\angle CAB$, then click OK. Use the same process to calculate cosine and tangent.

Your observations in Q2 give you a useful fact about right triangles. For any right triangle with a given acute angle, each ratio of side lengths has a given value, regardless of the size of the triangle. The three ratios you measured are called *sine*, *cosine*, and *tangent*.

- The sine, cosine, and tangent functions can be found on all scientific calculators, commonly abbreviated as sin, cos, and tan. Use Sketchpad's Calculator to calculate the sine, cosine, and tangent of $\angle CAB$. Match these calculations with the ratios they are equal to.

Trigonometric Ratios (continued)

Q3 Complete the ratios for cosine and tangent below.

$$\text{sine } \angle A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}}$$

$$\text{cosine } \angle A = \underline{\hspace{2cm}}$$

$$\text{tangent } \angle A = \underline{\hspace{2cm}}$$

Q4 Drag point C so that $\angle A$ measures as close to 30° as you can get it. Write approximate values for the sine, cosine, and tangent of 30° below. Use the definitions in Q3 and refer to the calculations in your sketch to find these values.

$$\sin 30^\circ = \underline{\hspace{1cm}} \quad \cos 30^\circ = \underline{\hspace{1cm}} \quad \tan 30^\circ = \underline{\hspace{1cm}}$$

Q5 Without measuring, figure out the measure of $\angle C$ and write down that number. Calculate the sine of that angle measure. The sine of $\angle C$ should be close to one of the trigonometric ratios for $\angle A$. Which one? Explain why this is so.

Q6 Drag point C and answer the following questions.

- What's the smallest possible value for the sine of an angle in a right triangle? What angle has this value? $\underline{\hspace{2cm}}$
- What's the greatest possible value for the sine of an angle in a right triangle? What angle has this value? $\underline{\hspace{2cm}}$
- Why can't you make $m\angle CAB$ exactly equal to 90° ?

d. Even though you can't make $m\angle CAB$ exactly equal to 90° , what do you think is the value of $\tan 90^\circ$? Explain.

e. For what angle is the tangent equal to 1? Why?

f. For what angle are the sine and cosine equal? Why?

g. Suppose an angle has measure x . Complete this equation:

$$\sin x = \cos \underline{\hspace{2cm}}.$$

Hint: Make \overline{AB} short so that you can drag point C up farther.